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Optimization of Interdigital Capacitors

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***Abstract*—The variation of Q and capacitance slope for series- and shunt-connected interdigital capacitors is shown. A theory suitable for interactive design of capacitors is given.**

I. Introduction

The use of interdigital capacitors on microstrip has become common place over the past few years [1], but little work has been published on the performance of the capacitors at X–J-band frequencies. Three factors must be considered when designing an interdigital capacitor, namely: capacitance slope, Q, and parasitics. This paper describes the effect of the interdigital capacitor layout on the first two factors.

II. Theory

Alley [2] has shown how a two-port matrix may be derived and used to represent a shunt-connected capacitor. This is the case where one side of the capacitor is grounded. Here the more general case of a series-connected capacitor, where both sides are used, is considered. In this case a four-port matrix is used to represent the capacitor and to determine the effect of changes in the capacitor shape.

Consider a pair of fingers of an interdigital capacitor (Fig. 1). Then a two-port admittance matrix $[Y_f]$ can represent the finger ports [3]. The elements of $[Y_f]$ are given by

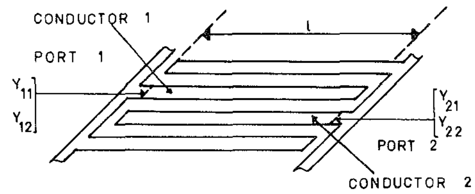

Fig. 1. Central form of an interdigital capacitor.

$$y_{11}=y_{22}=\frac{2[Zo_e\coth(\gamma_e l)+Zo_o\coth(\gamma_o l)]}{Zo_o^2+Zo_e^2+2Zo_oZo_e[\coth(\gamma_o l)\coth(\gamma_e l)+\operatorname{csch}(\gamma_o l)\operatorname{csch}(\gamma_e l)]} \quad (1)$$

$$y_{21}=y_{12}=\frac{-2[Zo_e\operatorname{csch}(\gamma_e l)-Zo_o\operatorname{csch}(\gamma_o l)]}{Zo_o^2+Zo_e^2+2Zo_oZo_e[\coth(\gamma_o l)\coth(\gamma_e l)+\operatorname{csch}(\gamma_o l)\operatorname{csch}(\gamma_e l)]} \quad (2)$$

where Zo_o and Zo_e are the odd- and even-mode impedances; γ_o and γ_e are the odd- and even-mode propagation constants; and l is the length of overlap of the fingers.

The impedances Zo_o and Zo_e were calculated using the computer program given by Smith [4] to compute the odd- and even-mode capacitances of the lines. Smith considers the case of a triplet of lines and obtains one pair of odd- and even-mode capacitances for the center line, and another pair for the outer lines. The values for the center lines were used to evaluate (1) and (2).

The loss components α_e,α_o for the propagation constants γ_o,γ_e were calculated using the formulas given by Schneider [5], but no allowance was made for surface roughness [6]. The formulas of Schneider assume a single-mode TEM transmission line. In order to use these formulas, it was assumed the coupled line loss components could be represented by two TEM-mode transmission lines of characteristic impedance Zo_o and Zo_e. A similar method was used by Horton [7] who shows the odd-mode component loss to be predominant. The result of these approximations is that the Q will take an optimistic value but show the correct variation for changes in shape of the interdigital capacitor.

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The two terminal strips connecting the fingers of the interdigital capacitor can be regarded as transmission lines shunted by a number of capacitors. The number of shunt capacitors being $(1/2)N_f$ for each terminal strip, where N_f is the number of fingers.

Assuming symmetry, the odd- and even-mode admittances for the terminal strip connections are given by

$$Y_{fo}=(y_{11}-y_{21})\tfrac{1}{2}N_f/l_c, \quad \text{odd} \tag{3}$$

$$Y_{fe}=(y_{11}+y_{21})\tfrac{1}{2}N_f/l_c, \quad \text{even} \tag{4}$$

where l_c is the length of the terminal strip.

Let Ce, Co, Le, Lo, Re, and Ro be the lumped parameters for the terminal strip before connection of the fingers. Then on connection of the fingers the admittances Y_{fo} and Y_{fe} will appear in parallel with the odd- and even-mode capacitances Co and Ce (Fig. 2). Thus at an angular frequency (ω) the characteristic impedance Zc and propagation constant γc can be resolved into odd- and even-mode components given by

$$Zc_e=\sqrt{\frac{Re+j\omega Le}{j\omega Ce+Y_{fe}}}, \quad \text{even mode} \tag{5}$$

$$\gamma c_e=\sqrt{(Re+j\omega Le)(j\omega Ce+Y_{fe})}\,. \tag{6}$$

The odd-mode expressions can also be derived.

The two terminal strips can now be regarded as parallel coupled lines and the corresponding four-port matrix derived [2]. Using the port configuration shown in Fig. 3, the impedance parameters are given by

$$Zp_{11}=Zp_{22}=Zp_{33}=Zp_{44}$$
$$=\tfrac{1}{2}[Zc_e\coth(\gamma c_e l_c)+Zc_o\coth(\gamma c_o l_c)] \tag{7}$$

$$Zp_{12}=Zp_{21}=Zp_{34}=Zp_{43}$$
$$=\tfrac{1}{2}[Zc_e\coth(\gamma c_e l_c)-Zc_o\coth(\gamma c_o l_c)] \tag{8}$$

$$Zp_{13}=Zp_{31}=Zp_{24}=Zp_{42}$$
$$=\tfrac{1}{2}[Zc_e\operatorname{csch}(\gamma c_e l_c)-Zc_o\operatorname{csch}(\gamma c_o l_c)] \tag{9}$$

$$Zp_{14}=Zp_{41}=Zp_{23}=Zp_{32}$$
$$=\tfrac{1}{2}[Zc_e\operatorname{csch}(\gamma c_e l_c)+Zc_o\operatorname{csch}(\gamma c_o l_c)]. \tag{10}$$

In the case of a center fed capacitor (Fig. 1) the system can be broken into three parts (Fig. 4). There are two open-circuit two-port admittance matrices $[Yp]$. The matrix $[Yp]$ is given by

$$[Yp]=[Zp]^{-1}.$$

Let Nz be the number of fingers across the feeder. Then a new two-port matrix $[Yz]$ can be formed with the elements

$$Yz_{11}=Yz_{22}=2Yp_{11}+Nzy_{11} \tag{11}$$

$$Yz_{12}=Yz_{21}=2Yp_{12}+Nzy_{12}. \tag{12}$$

From this matrix $[Yz]$, a Π equivalent circuit for the capacitor can be found.

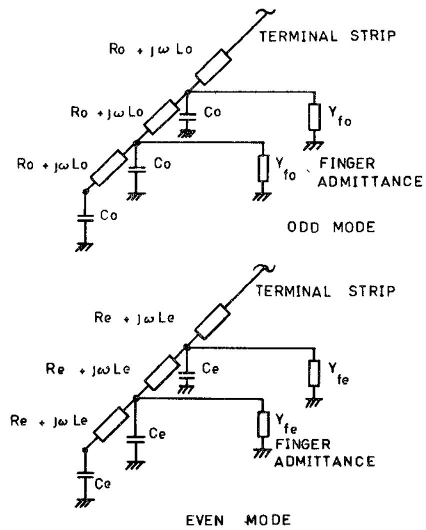

Fig. 2. The manner of connection of the fingers to the terminal strip.

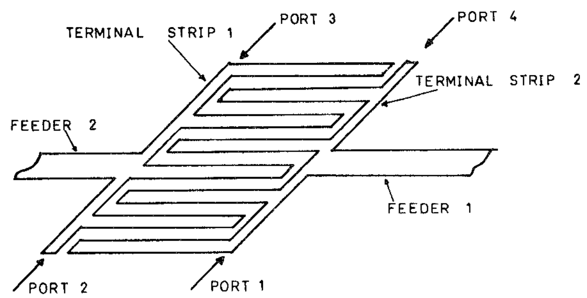

Fig. 3. Port configuration for the terminal strips.

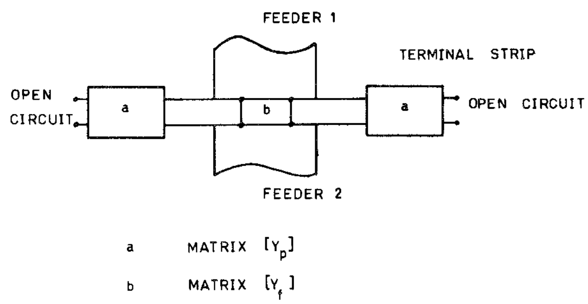

Fig. 4. Connection of the terminal strips to the feeders.

III. Results

Equations (11) and (12) were used to compute the theoretical variation in Q and capacitance slope of different series and shunt capacitors to determine the effect of varying the number of fingers, for a constant capacitance.

Fig. 5(a) and (b) show the variation of Q for a capacitance of 0.3 pF at 10 GHz with the following parameters:

substrate relative permittivity 9.6;
line thickness 2.8 μm;
terminal strip width 50 μm;
substrate thickness 625 μm;
feeder width 625 μm.

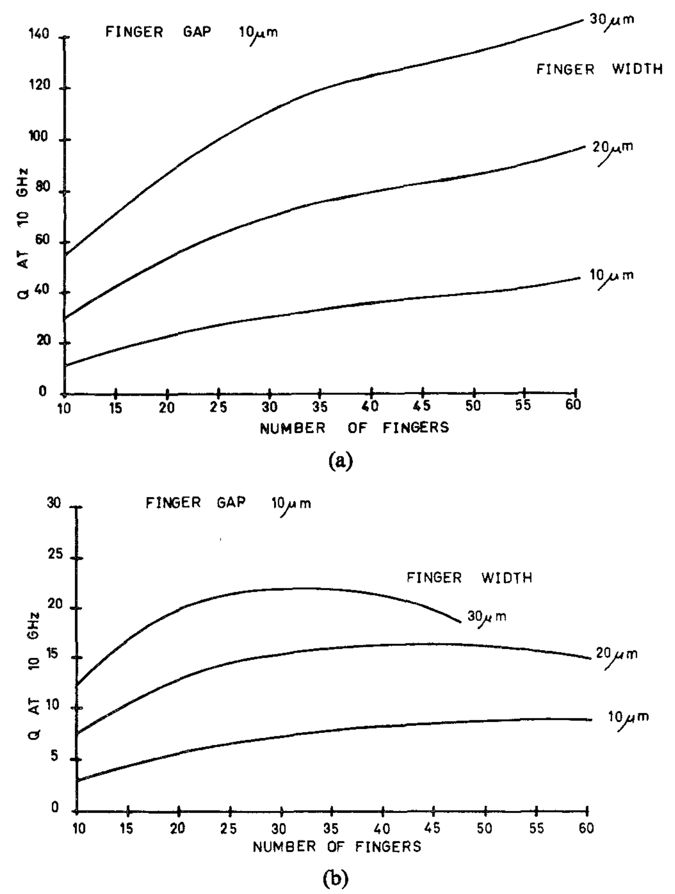

Fig. 5(a). The Q of a 0.3-pF shunt connected capacitor at 10 GHz. (b) The Q of a 0.3-pF series connected capacitor at 10 GHz.

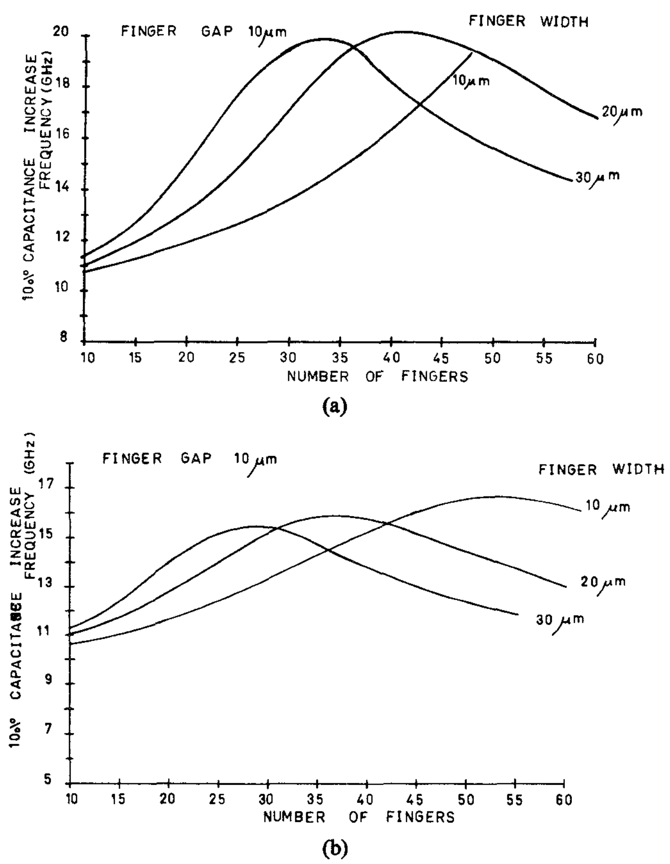

Fig. 6(a). The 10-percent capacitance increase frequency of a 0.2-pF shunt connected capacitor. (b) The 10-percent capacitance increase frequency of a 0.2-pF series connected capacitor.

For each number of fingers the finger length was chosen to keep the capacitance constant.

In the shunt capacitor case, the Q reaches a maximum when the finger length is zero. The larger the number of fingers the shorter the fingers are for a constant capacitance. Thus in the limit a shunt capacitor is simply two open-circuit stubs in parallel.

The series capacitor case is different in that a clear optimum exists. The difference in results being due to the capacitance to the ground plane. In the series case this capacitance is unwanted but in the shunt case the ground capacitance and finger coupling capacitance act in parallel.

The Q increases in both cases when the finger width is increased due to the presence of more metal in the structure causing the even-mode impedance to be lower. However, wider fingers means a larger capacitance to ground which is undesirable in the series-connected case.

Comparison of the computed Q values with the values quoted for a 0.25-pF interdigital capacitor by Pengelly [1] show a Q value of 40 to be expected with a metallization thickness of 3 μm. Q values in this region are predicted.

The variation of the capacitance slope with the number of fingers is shown in Fig. 6(a) and (b). Here, f (10 percent) is defined as the frequency at which the capacitance has increased by 10 percent on the designed capacitance. The capacitor with minimum capacitance slope has the maximum f (10 percent). It can be seen an optimum condition exists for both series and shunt capacitors.

This variation in f (10 percent) arises from the change of the series resonant frequency of the fingers and the terminal strip. For a constant capacitance, an increase in the number of fingers means a decrease in finger length which raises the series resonant frequency. Conversely, an increase in the capacitor width causes a lowering of the series resonant frequency of the terminal strip.

Comparing Figs. 5 and 6 it can be seen that a compromise has to be made between maximum Q and maximum f (10 percent).

IV. Conclusion

The theory has been presented in a form usable for interactive design of interdigital capacitors. The effect of the capacitor shape on Q and capacitance slope has been shown.

Acknowledgment

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Measurement of Dielectric Parameters at Microwave Frequencies by Cavity-Perturbation Technique

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***Abstract*—Relations for evaluating dielectric parameters from changes in resonance frequency and Q of a cylindrical TM_{010}-mode cavity have been derived for thin samples of length less than the height of the cavity. Although derived under some simplifying assumptions, they reduce to the standard form when the length of the specimen equals the height of the cavity, and yield consistent results when applied to different lengths of the same material.**

I. Introduction

THE CAVITY perturbation technique has been extensively employed for studying the dielectric and magnetic properties of materials at microwave frequency [1]–[4]. The method has its own advantages but it has certain limitations, especially the requirement that the volume of the specimen must be small so as to produce a negligible effect on the field configuration in the cavity. Also the sample must be in one of the specified shapes for which formulas have been worked out. For the case of a cylindrical cavity resonating in the TM_{010} mode, the sample used is mostly in the form of a thin rod, the length of which equals the height of the cavity, so that both the ends of the specimen are in contact with the cavity walls [5]. The quality factor Q of the cavity depends upon its height and it is convenient to work with high-Q cavities. However, often it becomes difficult to machine long and thin samples. A need, therefore, arises for extending the cavity-perturbation technique to include specimens of length less than the height of the cavity. In this article a set of formulas, based on certain assumptions, have been developed by calculating an effective depolarizing factor in order to meet this requirement. The resulting equations for dielectric parameters have been found to be adequate in yielding consistent results when applied to specimens of different lengths of the same material. Moreover, these formulas readily reduce to the ones used for full-size specimen under the condition that the length of the specimen is equal to the height of the cavity.

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II. Theory

A. Relations for a Prolate Ellipsoid of Insufficient Length Held Coaxially in a Cylindrical Cavity Resonating in the TM_{010} Mode

Waldron [5], [6] has shown that relative change in the complex resonance frequency due to the insertion of a small specimen in a resonating cavity bounded by perfectly conducting surface is given by

$$\left[\frac{\delta\Omega}{\omega_0}\right] = \frac{\iiint_{V_0}\{(\vec{E}_1\cdot\vec{D}_0 - \vec{E}_0\cdot\vec{D}_1) - (\vec{H}_1\cdot\vec{B}_0 - \vec{H}_0\cdot\vec{B}_1)\}\,dV}{\iiint_{V_0}\{\vec{E}_0\cdot\vec{D}_0 - \vec{H}_0\cdot\vec{B}_0\}\,dV} \quad (1)$$

where $[\delta\Omega/\omega_0]$ is related to the changes in the resonance